Article

Service Good as an Intermediate Input and Optimal Government Policy in an Endogenous Growth Model

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Abstract

The present article considers an endogenous growth model in which the service output is used as intermediate good in commodity sector, tax is imposed on manufacturing product and the revenue earned is invested to create human capital. It is shown that there exists a unique, saddle path stable steady-state growth rate of human capital accumulation and a unique growth-maximizing tax rate. The optimal tax rate for the command economy is compared with growth-maximizing tax rate in competitive economy. A numerical analysis shows that the command economy will have a higher growth rate than the competitive economy. An extension of the model where households privately spend for accumulation of human capital yields the same growth rate as that of the command economy of the previous model.

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Introduction

The last few decades have experienced a rapid growth in the service sector, which has been reflected in an upward trend in the usage of service goods as consumption good as well as producer good (as intermediate input in production). The present study focuses on the service good as a producer good. According to Ishikawa (1992), producer services that have expanded with economic growth include business and professional services (such as management consulting, engineering consulting and data processing), financial and insurance services and real estate services. Banga and Goldar (2007) observed that in case of India, contribution of services input to output and productivity growth in manufacturing (organized) has increased substantially in the 1990s. Francois and Woerz (2008) pointed out that the importance of services in production as input rises with the level of development of an economy specially after the information technology revolution. Further, using a cross-country panel data analysis, they have found the evidence that the producer services (business services/communication services/financial services/insurance services/transportation services) are significantly present in the production of food, textile, leather, clothing, wood, paper, coke, chemicals, machinery, motor vehicles and electrical equipment. Das and Saha (2015) also showed that in the USA, the UK and Japan, the share of pure business services in the services sector as a whole has nearly or more than doubled in a span of over three decades 1970–2006. Behuria and Khullar (1994) documented the role of intermediate services in economic development in the context of Malaysia. They have observed that services are increasingly used as an input of production in various sectors. Thus, there are many empirical evidences showing the increasing importance of producer services in the production process of different goods.

Ishikawa (1992) theoretically developed a model where producer service is used as an intermediate good to the manufacturing sector, while service is produced using labour alone with constant returns to scale (CRS) technology. Ishikawa (1992) worked on a simple dynamic small open economy model, which allows for changes in both industrial structure and trade patterns during the process of economic growth. In this article, he has considered three sectors: manufacturing, agriculture and producer service, where producer service is an intermediate good in the manufacturing sector. In this model, learning by doing in the service sector is the source of endogenous growth, and whether the economy will flourish or not depends on the initial productivity of labour in the producer service sector. Thus, the producer service sectors play a crucial role in the model. Another theoretical model by Das and Saha (2015) also considers service as an intermediate input to explain the fast growth of service sector, which is sometimes greater than the manufacturing sector also. The article develops a two-sector closed economy model with manufacturing and service sectors. The analysis focuses on business services, while household services are also considered. The manufacturing output is produced with human capital and business service, and the service output solely depends on the human capital. It is argued that differences in returns to scale between the two sectors and employment frictions in manufacturing explain why the growth rate of the service sector may be higher. The model also features that within the service sector, the business services sub-sector may grow faster than the household services. The producer services are human capital intensive. As a result, contribution of education towards economic growth has gradually increased in recent years. Bosworth, Collins, and Virmani (2006) showed that in case of India, service sector has registered the largest improvements in the educational attainment of its workforce.[1]

Given the importance of the service good as an input to manufacturing sector, the present study hypothetically considers a two-sector endogenous growth model where service output is used as an input in the commodity manufacturing sector.[2] We consider human capital as one of the most important factors in producing service output.[3] We first assume

[1] The papers by Riley (2014), Lucas (1988), Mankiw, Romer, and Weil (1992), Fuente and Doménech, (2006), and so on discuss various issues related to human capital and service goods.

[2] As already mentioned, Francois and Woerz (2008) found a number of evidence that pointed towards using service good as an input in various industries. Apart from that, we can consider a simple dressmaker who uses sewing machine and tailoring services for production purpose. Alternatively, the fully automated electricity plants use machineries and engineering services to generate electricity.

[3] The papers by Moro (2007), Sekar, Delgado, and Ulu (2015), Barua and Pant (2014), Imbruno (2014), Miroudot, Lanz, and Ragoussis (2009), Psarianos (2002), and so on have discussed the matters related to various aspects of intermediate goods other than service good in the context of closed or open economies.

public education system, and hence, human capital is generated through government expenditure on education. This model is considered as the basic model in our article. In addition, the article develops the command economy version of the basic model to consider the issues of optimal taxation in the presence of producer services. Finally, we build up another model where household allocates a fraction of their income for human capital investment. This version of the model does not incorporate government sector like the other version. This analysis helps us to compare the features of growth process of the competitive economy under government intervention vis-à-vis the case where government does not interfere.

Contribution of this article is to incorporate public education system for human capital formation when human capital is used to generate producer services.[4] The article contributes to existing literature by considering the issue of optimal taxation in the presence of producer services in the endogenous growth model. After the emergence of goods and service tax, many countries, such as Canada, France, the UK, New Zealand, Malaysia, Singapore, India and many more, are still struggling to rationalize the tax rate structure. A comparison of tax rates by countries is difficult as tax laws in most countries are extremely complex, and the tax burden falls differently on different groups in each country and subnational unit. Different tax rates are levied on goods and services in different countries. Thus, the article contributes to the literature by finding out optimal and growth-maximizing tax policy in the competitive and command economy where service sector output is used as an input in the manufacturing sector. Given the present scenario of service-led growth, our article contributes to the literature by linking the issue of public expenditure-led education to the issue of service sector's contribution to economic growth. Next, we compare this to the situation where human capital formation is undertaken privately by the households in the presence of producer services. To best of our knowledge, we did not come across any paper that has done this exercise.

[4] The spending of government expenditure on public resources, through taxation to create both human capital and physical capital, in endogenous growth models has also been analysed in various studies, such as by Garcia-Castrillo and Sanso (2000), Gomez (2003), Futagami, Morita, and Shibata (1993), Faig (1995), Dasgupta (1999), Fernández, Novales, and Ruiz (2004), Woo (2005), Tsoukis and Miller (2003), Chen and Lee (2007), Hollanders and Weel (1999), Greiner (2006), and so on. However, these models did not include the issue of service good as an intermediate product.

We find some interesting results. The results show that under competitive economy framework, in the basic model where government imposes tax on commodity sector to finance human capital accumulation that is used in the production of producer service, the growth rate depends on the rate of taxation, and a unique growth rate maximizing tax rate will exist. This growth rate is saddle path stable. However, the existence of tax on commodity in the competitive market is creating a distortion in the model. Further, a numerical analysis shows that the command economy will have a higher growth rate than the competitive economy. The first best solution is obtained when households privately spend for accumulation of human capital. The command economy growth rate, after endogenizing the tax rate, is found to be the same as the first best solution.

The rest of the article is organized as follows: in the second section, the basic model is presented; in sub-section 'Households, Firms and Government' assumptions of the model are described. Sub-section 'Decentralized Economy: The Basic Model' presents the basic model in competitive economy framework. Sub-section 'Command Economy: The Basic Model' presents the command economy version of the basic model. The third section presents an extension where households privately invest for human capital formation. The comparison of tax rates for basic model under competitive and command economy framework is done with a numerical analysis in the fourth section. Finally, the fifth section concludes the article.

Basic Model

This section discusses the basic assumptions of the model and derives the growth path under competitive and command regime.

Households, Firms and Government

This article considers a hypothetical closed economy model with two sectors, namely commodity sector and service sector. The service sector produces producer service, which is exclusively used as an intermediate input in producing commodity output. The total labour force is homogeneous as far as skill is concerned. Identical rational agents inhabit the economy. Production technology is subject to CRS. The household sector chooses the path of per capita consumption of commodity output by maximizing the present discounted value of utility over the infinite time horizon, ρ being the discount rate and σ being the elasticity of marginal utility; inverse of σ is known as inter-temporal elasticity of substitution. N represents the total labour force or working population. Preferences over consumption are given by the following function where 'c' denotes flow of real per capita consumption of commodity output:

$$u(c) = \int_0^\infty \frac{(c^{1-\sigma} - 1)}{(1-\sigma)} e^{-\rho t} N(t) dt \tag{1}$$

Here, we assume that the output in the commodity sector can be used for consumption or investment purposes. The commodity output is produced using physical capital and producer service good. The producer service output is a function of human capital and physical capital. Both the production functions are Cobb–Douglas type. Here 'K' stands for the level of physical capital. Let α and β be the commodity output elasticity of physical capital and service output elasticity of skilled labour, respectively. The commodity and service output production functions can be written as:

$$Y_C = A\{(1-\varphi)K\}^{\alpha} Y_s^{1-\alpha} \tag{2}$$

$$Y_s = B(Nh)^{\beta}(\varphi K)^{1-\beta} \tag{3}$$

where Y_c and Y_s are the flow of commodity output and service output, respectively. It is obvious that $(1 - \alpha)$ measures the commodity output elasticity of service product. Similarly, $(1 - \beta)$ measures the service output elasticity of physical capital. The level of population is growing at an exponential rate in the following manner:

$$N(t) = N_0 e^{nt} \tag{4}$$

where N_0 is the population size at initial time period. It is assumed that the initial amount of population $N_0 = 1$. Further, we assume that the general skill level of a worker is h. Skill is accumulated through education. The aggregate effective skilled labour input in commodity production is Nh. Let φ be the fraction of physical capital that is allocated to the service sector. The remaining $(1-\varphi)$ fraction is engaged in producing commodity output. In the present section, it is assumed that government spends money on working population to create human capital. We did not assume any allocation of time between production and skill accumulation by the skilled individual. Hence, in this model, an individual simultaneously works and accumulates skill. Skill accumulation may be assumed to take place through government sponsored on job training of workers or apprenticeship programmes.

Following Beauchemin (2001), Cardak (2004), Tanaka (2003), Glomm and Ravikumar (2001), we assume per capita government expenditure as an input in the production process of human capital in the basic model. The findings by Blankenau, Simpson, and Tomljanovich (2007) based on pooled data from 1960 to 2000 for 80 countries support a positive relationship between public education expenditure and human capital formation in developed countries. In most of the developing countries too, governments play key roles in fostering education through providing free primary education, highly subsidized secondary education, research funding and student financial assistance. Because of the 'non-rival' nature of the skill, it is assumed that there is no diminishing return to G in skill accumulation.

The human capital accumulation can be written as:

$$\dot{h} = \eta \frac{G}{N} \tag{5}$$

Here, η is the technology parameter of human capital accumulation and G is the government expenditure on education. It is further assumed that only the commodity sector is being taxed. Let the tax rate be τ which is levied on per unit of commodity output. Even in the study of Futagami et al. (1993), the accumulation rate of public capital is proportional to the tax revenue or equivalently government expenditure.[5]

The balanced budget equation can be written as follows:

$$\mathrm{G} = \mathrm{T} = \tau \mathrm{Y}_C \tag{6}$$

A part of disposable income is consumed, and the rest is invested to form physical capital. Hence, the physical capital accumulation function is given by

$$\dot{K} = (1-\tau) Y_C - cN = (1-\tau)(rK + wNh) - cN \tag{7}$$

[5] If instead of assuming direct government expenditure as an input in human capital accumulation, we assume that government expenditure is used to finance the wage rate of the specialized labour (teachers) used to generate human capital, endogenous growth rate is obtained, but growth maximizing tax rate cannot be found out.

In the decentralized economy, the households own all capital. It can be easily verified that $Y_C = rK + wNh$.[6] The final product is produced by a representative firm that maximizes profit. The objective of the economy is to maximize the present discounted value of utility over the infinite time horizon defined by Equation (1) subject to the wealth accumulation constraint. The next section presents the competitive economy equilibrium.

Decentralized Economy: The Basic Model

The objective of an individual consumer is to maximize present discounted value of utility over the time horizon choosing the consumption path. The current value Hamiltonian for this particular problem is given as follows:

$$H = \frac{c^{1-\sigma} - 1}{(1-\sigma)} N(t) + \theta[(1-\tau)(rK + wNh) - cN] \tag{8}$$

In competitive economy, a representative household chooses c, the flow of consumption. So, c is the decision variable, K is a state variable and θ is the shadow price of physical capital. While solving this Hamiltonian function, tax rate τ is considered to be given as per the decentralized regime.

The first-order optimality conditions for maximization of Hamiltonian is given by

$$c^{-\sigma} = \theta \tag{9}$$

The equation of motion of co-state variable is given by

$$\frac{\dot{\theta}}{\theta} = \rho - (1-\tau) r \tag{10}$$

The profit of the producers for the commodity sector and service is:

$$\pi_c = p_c Y_C - r(1-\varphi)K - p_s Y_s \tag{11}$$

$$\pi_s = p_s Y_s - (r\varphi K) - (wNh) \tag{12}$$

[6] For proof, see Appendix A.

Here, r is the rate of interest, w is the real wage rate and p_s is the per unit price of service output. It is assumed that the commodity output is numeraire commodity, which implies that per unit price of commodity output, p_c, is unity.

Both the output and the factor markets are characterized by perfect competition. Hence, equating the value of the marginal product of each factor input to its return and using profit maximization condition, we get the following expressions:

From profit maximization of final consumption good sector

$$r = A\alpha((1-\varphi)K)^{\alpha-1}Y_s^{1-\alpha} \tag{13a}$$

$$p_s = A((1-\varphi)K)^{\alpha}(1-\alpha)Y_s^{-\alpha} \tag{14}$$

From profit maximization of producer service sector

$$r = p_s B(hN)^{\beta}(1-\beta)(\phi K)^{-\beta} \tag{13b}$$

which equivalently in intensive form is

$$r = \alpha A(1-\hat{\varphi})^{\alpha-1}B^{1-\alpha}\hat{\varphi}^{(1-\beta)(1-\alpha)}k^{-\beta(1-\alpha)} \tag{13c}$$

$$p_s = A((1-\varphi)K)^{\alpha}(1-\alpha)Y_s^{-\alpha} = AB^{-\alpha}(1-\phi)^{\alpha}(1-\alpha)\phi^{\alpha(\beta-1)}k^{\alpha\beta}$$

$$w = p_s B\beta(Nh)^{\beta-1}(\phi K)^{1-\beta} = p_s B\beta(\phi k)^{1-\beta} \tag{15}$$

Here k denotes the physical capital per unit of skill and is defined by $k = \frac{K}{hN}$. From the aforementioned system of equations using no arbitrage condition and equating (13a) and (13b), the value of φ is found as follows:

$$\hat{\varphi} = \frac{(1-\alpha)(1-\beta)}{1-\beta(1-\alpha)} \tag{16}$$

The steady-state growth paths in market economy is defined as the path along which c, h, K grow at constant rate, and the value of φ is time-independent. The growth rate of human capital accumulation and that of per unit commodity output consumption and the growth rate of physical capital are given by

$$\gamma_h^{comp} = \eta\tau A\{(1-\hat{\varphi})\}^{\alpha} B^{1-\alpha} (\hat{\varphi})^{(1-\beta)(1-\alpha)} k^{1-\beta(1-\alpha)} \tag{17}$$

$$\gamma_c^{comp} = \frac{(1-\tau)\alpha A(1-\hat{\varphi})^{\alpha-1} B^{1-\alpha} \hat{\varphi}^{(1-\beta)(1-\alpha)} k^{-\beta(1-\alpha)} - \rho}{\sigma} = \frac{(1-\tau)r-\rho}{\sigma} \tag{18}$$

$$\gamma_K^{comp} = n + \gamma_h^{comp} \tag{19}$$

where γ_x stands for growth rate of the variable x.

Along the steady state, $\gamma_c = \gamma_h$. Equating γ_c, γ_h from Equations (17) and (18), the following equation is obtained in terms of k, τ and other parameters. τ is considered to be given in the competitive economy.

$$\sigma\eta\tau A\{(1-\hat{\varphi})\}^{\alpha} B^{1-\alpha} (\hat{\varphi})^{(1-\beta)(1-\alpha)} k^{1-\beta(1-\alpha)} + \rho = (1-\tau)\alpha A(1-\hat{\varphi})^{\alpha-1} B^{1-\alpha} \hat{\varphi}^{(1-\beta)(1-\alpha)} k^{-\beta(1-\alpha)} \tag{20}$$

Let $f(k) = L.H.S$ of Equation (20), where $f' > 0$. $f(k)$ is depicted by I–J curve in Figure 1. $g(k) = R.H.S$ of Equation (20), where $g' < 0$. $g(k)$ is depicted by LM in Figure 1.

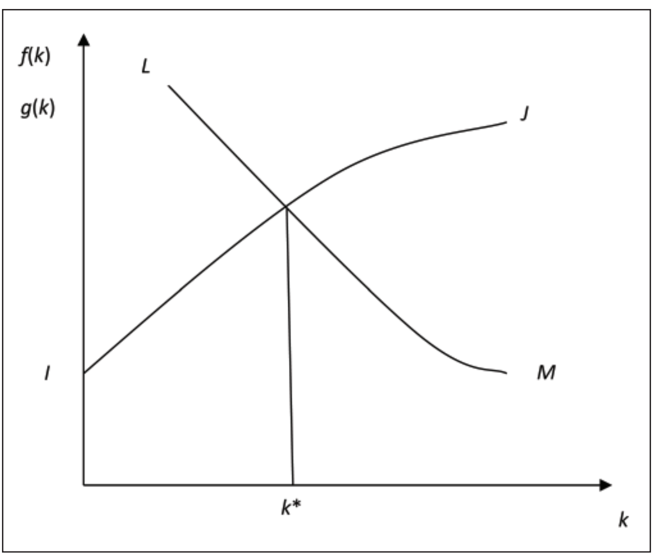

Figure 1. Determination of Equilibrium k^* in Decentralized Economy

Source: The authors.

Diagrammatically, it is shown that there exists unique value of k that satisfies Equation (20) and that can be found out in terms of τ and other parameters.

In Figure 1, *LM* and *IJ* represent $g(k)$ and $f(k)$ functions from Equation (20), respectively. The equilibrium value of k is given by k^* in the figure.

In Figure 1, k^* is the equilibrium value for k in competitive economy. Hence, there exists a unique solution of k, given tax rate τ in competitive framework. Consequently, the growth rates of human capital, commodity output and that of physical capital can be solved endogenously.

Proposition 1: *There exists positive, unique steady-state growth rate for human capital, physical capital and production and consumption of commodity output in competitive economy.*

It is found that

$$\frac{dk}{d\tau} = -\frac{[\alpha + \sigma\eta k]}{[\sigma\eta\tau(1-\beta(1-\alpha)) + (1-\tau)\alpha\beta(1-\alpha)k^{-1}]} < 0 \tag{21}$$

Thus, with an increase in the tax rate, the ratio of physical capital to human capital decreases. An increased tax rate implies enhanced government expenditure for human capital accumulation, and human capital grows more rapidly. On the other hand, as the tax is imposed on commodity output, it hampers physical capital accumulation. Thus, increased tax rate implies lower growth of production and consumption of commodity. As a result, the value of physical capital that is allocated per head human capital, that is, k falls for an increase in τ.

Differentiating Equations (17) and (18) with respect to tax rate gives the following results:

$$\frac{\partial \gamma_h}{\partial \tau} = \frac{\eta A\{(1-\hat{\varphi})^{\alpha} B^{1-\alpha}(\hat{\varphi})^{(1-\beta)(1-\alpha)}\} k^{(1-\beta(1-\alpha))}(\beta(1-\alpha)-\tau)}{[\sigma\eta\tau(1-\beta(1-\alpha))k + (1-\tau)\alpha\beta(1-\alpha)]} \tag{22}$$

$$\frac{\partial \gamma_c}{\partial \tau} = \frac{\alpha A\{(1-\hat{\varphi})\}^{\alpha-1} B^{1-\alpha}(\hat{\varphi})^{(1-\beta)(1-\alpha)}\} k^{(1-\beta(1-\alpha))}\eta(\beta(1-\alpha)-\tau)}{[\sigma\eta\tau(1-\beta(1-\alpha))k + (1-\tau)\alpha\beta(1-\alpha)]} \tag{23}$$

From the aforementioned equation, we find that the relationship with growth rates and τ will depend on the tax rate. Further, the growth rates are maximized for the tax rate $\tau^* = \beta(1-\alpha)$.[7] This tax rate is

[7] Second-order conditions for maximization are checked and satisfied.

equal to indirect elasticity of human capital in the production of final consumption good.

Proposition 2: *There exists a unique growth-maximizing tax rate for the competitive economy.*

It is also found that

$$\frac{dk}{d\eta} = -\frac{\sigma\tau k}{[\sigma\eta\tau(1-\beta(1-\alpha)) + (1-\tau)\alpha\beta(1-\alpha)k^{-1}]} < 0 \quad (24)$$

From Equation (18), we see that γ_c is negatively related to k. Since k is negatively related to η, the steady-state growth rate of consumption is positively related to η. As the technological efficiency of education sector increases, per capita physical capital–human capital ratio in steady state decreases, but the growth rate of consumption, human capital and aggregate physical capital increases. This is obvious because as education sector becomes more efficient, it can generate more human capital. Hence, the ratio of per capita physical capital to human capital declines.

A rise in technological efficiency of education sector will boost up the growth of human capital. This will in turn raise the service output; as a result, commodity output will also experience a rise in growth rate.

Stability Analysis of the Basic Model

Dividing both sides of Equation (7) by K, we get

$$\gamma_K = \frac{\dot{K}}{K} = (1-\tau)(\frac{y_c}{K}) - \frac{cN}{K}$$

Using Equations (2), (3) and definition of $k = \frac{K}{hN}$, we have

$$\frac{\dot{K}}{K} = B^{(1-\alpha)}(1-\tau)A(1-\varphi)^{\alpha}k^{-\beta(1-\alpha)}\varphi^{(1-\alpha)(1-\beta)} - (\frac{c}{kh})$$

Let $(\frac{c}{h}) = x$

Using Equations (2), (3), (5) and (6), we derive

$$\gamma_h = \frac{\dot{h}}{h} = \eta A\tau(1-\varphi)^{\alpha}B^{(1-\alpha)}\varphi^{(1-\beta)(1-\alpha)}k^{1-\beta(1-\alpha)}$$

Now, $\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{h}}{h} - n$

Hence,

$$\dot{k} = B^{(1-\alpha)}(1-\tau)A(1-\varphi)^{\alpha}k^{1-\beta(1-\alpha)}\varphi^{(1-\alpha)(1-\beta)} - (\frac{c}{h}) - \eta A\tau(1-\varphi)^{\alpha}B^{(1-\alpha)}\varphi^{(1-\beta)(1-\alpha)}k^{2-\beta(1-\alpha)} - nk \quad (25)$$

From Equation (18), we have

$$\dot{c} = \frac{c}{\sigma}[(1-\tau)A\alpha(1-\varphi)^{\alpha-1}B^{1-\alpha}k^{-\beta(1-\alpha)}\varphi^{(1-\beta)(1-\alpha)} - \rho] \quad (26)$$

As $(\frac{c}{h}) = x$

$\frac{\dot{c}}{c} - \frac{\dot{h}}{h} = \frac{\dot{x}}{x}$

Therefore, the system of dynamic equation of the present model is as follows:

$$\dot{x} = x[\{\frac{(1-\tau)\alpha A(1-\varphi)^{\alpha-1}B^{1-\alpha}\varphi^{(1-\beta)(1-\alpha)}k^{-\beta(1-\alpha)} - \rho}{\sigma}\} - \eta A\tau(1-\varphi)^{\alpha}B^{(1-\alpha)}\varphi^{(1-\beta)(1-\alpha)}k^{1-\beta(1-\alpha)}] \quad (27)$$

$$\dot{k} = AB^{(1-\alpha)}(1-\tau)(1-\varphi)^{\alpha}k^{1-\beta(1-\alpha)}\varphi^{(1-\alpha)(1-\beta)} - x - \eta A\tau(1-\varphi)^{\alpha}B^{(1-\alpha)}\varphi^{(1-\beta)(1-\alpha)}k^{2-\beta(1-\alpha)} - nk \quad (28)$$

If profit-maximizing conditions and no arbitrage conditions are assumed to be satisfied at every point of time, φ is always a constant. We are not getting any dynamics of φ.

At steady state, $\dot{k} = 0$. Hence, from Equation (28), we get

$$x = AB^{(1-\alpha)}(1-\tau)(1-\varphi)^{\alpha}k^{1-\beta(1-\alpha)}\varphi^{(1-\alpha)(1-\beta)} - \eta A\tau(1-\varphi)^{\alpha}B^{(1-\alpha)}\varphi^{(1-\beta)(1-\alpha)}k^{2-\beta(1-\alpha)} - nk \quad (29)$$

Now, $\frac{dx}{dk} = 0$ at $k = \hat{k}$ and $\frac{d^2x}{dk^2} < 0$. Hence, x curve given by Equation (29) is maximized at $k = \hat{k}$. In Equation (27), setting $\dot{x} = 0$, we get a fixed value of $k = k^*$.We can represent $\dot{x} = 0$ and $\dot{k} = 0$ curves in x–k plane and graphically illustrate the transitional dynamics.

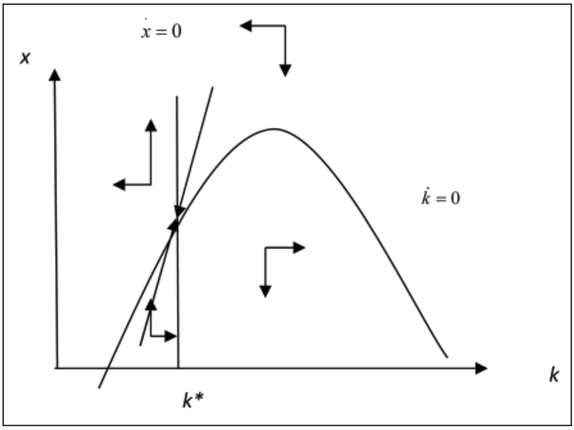

Figure 2. Stability Analysis of the Basic Model
Source: The authors.

From the above figure, we observe that like the Ramsey model, the steady state obtained in this model is saddle path stable.

Proposition 3: *The equilibrium growth rate is saddle path stable.*

Command Economy: The Basic Model

In this section, we discuss the same problem in command economy set up. There are no external effects and imperfect competition in this model. But, in the competitive economy, individual considers tax rate as given, and thus, individual agents consider human capital accumulation function as given while maximizing the present discounted value of utility. In the command economy, the social planner takes into account the human capital accumulation function while optimizing the welfare of the society, deciding tax rate and allocating resources optimally. The objective of the social planner is to maximize the present discounted value of utility defined by Equation (1) subject to the constraints given by Equations (5)–(7). The Hamiltonian function is given by the following equation:

$$H = \frac{N}{(1-\sigma)}(c^{1-\sigma} - 1) + \theta_1[(1-\tau)A\{(1-\varphi)K\}^{\alpha}B^{1-\alpha}(Nh)^{\beta(1-\alpha_1)} (\varphi K)^{(1-\beta)(1-\alpha)} - cN] + \theta_2[\eta\frac{\tau}{N}A\{(1-\varphi)K\}^{\alpha}B^{1-\alpha}(Nh)^{\beta(1-\alpha)} (\varphi K)^{(1-\beta)(1-\alpha)}] \quad (30)$$

where θ_1 and θ_2 are the shadow prices associated with $\dot{K}$ and $\dot{h}$ which stand for physical investment and human capital accumulation. Here, the decision variables of the social planner are c, ϕ, τ and the state variables are K and h. From the first-order conditions, the growth rates of commodity output production, human capital and physical capital are solved.

The first-order optimality conditions are as follows:

$$c^{-\sigma} = \theta_1 \tag{31}$$

$$\frac{dH}{d\varphi} = 0$$

or

$$\begin{aligned}&\{(1-\beta)(1-\alpha)\varphi^{(1-\beta)(1-\alpha)-1}(1-\phi)^{\alpha}\\&+\alpha(1-\phi)^{\alpha-1}(-1)\varphi^{(1-\beta)(1-\alpha)}\}\\&[AK^{\alpha}B^{1-\alpha}(Nh)^{\beta(1-\alpha)}K^{(1-\beta)(1-\alpha)}][\theta_1(1-\tau)+\theta_2\eta\frac{\tau}{N}] = 0\end{aligned} \tag{32}$$

$$\frac{dH}{d\tau} = 0$$

or

$$\frac{\theta_1}{\theta_2} = \frac{\eta}{N} \tag{33}$$

The equations of motion of co-state variables are as follows:

$$\begin{aligned}\frac{\dot{\theta}_1}{\theta_1} &= \rho-(1-\tau)A(1-\varphi)^{\alpha}\{1-\beta(1-\alpha)\}B^{1-\alpha}(Nh)^{\beta(1-\alpha)}\\&\varphi^{(1-\beta)(1-\alpha)}K^{\{\alpha+(1-\beta)(1-\alpha)-1\}}\\&+\frac{\theta_2}{\theta_1}\eta\frac{\tau}{N}A(1-\varphi)^{\alpha}B^{(1-\alpha)}(Nh)^{\beta(1-\alpha)}\varphi^{(1-\beta)(1-\alpha)}\\&\{1-\beta(1-\alpha)\}K^{\{\alpha+(1-\beta)(1-\alpha)-1\}}\end{aligned} \tag{34}$$

$$\begin{aligned}\frac{\dot{\theta}_2}{\theta_2} &= \rho-A(1-\varphi)^{\alpha}B^{1-\alpha}\beta(1-\alpha)h^{\beta(1-\alpha)-1}\\&\varphi^{(1-\beta)(1-\alpha)}(\frac{K}{Nh})^{1-\beta(1-\alpha)}\end{aligned} \tag{35}$$

Steady-State Growth Path Under Command Economy: The Basic Model

From the first-order optimality conditions, we derive the steady-state growth rates. The growth rate of per capita commodity output is given by

$$\gamma_c = \frac{\{1-\beta(1-\alpha)\}A(1-\varphi)^{\alpha}B^{1-\alpha}\varphi^{(1-\beta)(1-\alpha)}k^{-\beta(1-\alpha)}-\rho}{\sigma} \tag{36}$$

The growth rate of human capital accumulation is given by the following equation:

$$\gamma_h = \eta\tau A(1-\varphi)^{\alpha}B^{1-\alpha}(\varphi)^{(1-\beta)(1-\alpha)}k^{1-\beta(1-\alpha)} \tag{37}$$

Since along the steady state as γ_c is constant (followed from Equation (36)), k is also constant.

Therefore, $\gamma_k = 0$. Here, $k = \frac{K}{Nh}$. Taking logarithm of both sides of the aforementioned equation,

$\gamma_k = \gamma_K - n - \gamma_h$. As $\gamma_k = 0$, the growth rate of aggregate physical capital in steady state is given by $\gamma_K = \gamma_h + n$

The fraction of the physical capital that is allocated to produce service output is

$$\phi_{command} = \frac{(1-\beta)(1-\alpha)}{1-\beta(1-\alpha)} \tag{38}$$

Note that φ in command economy is the same as that obtained in competitive economy. The reason of the aforementioned finding is the absence of imperfect competition or any kind of external effects in this model. Using the value of $\phi_{command}$ and Equation (13c) in Equation (36), we get $\gamma_c = \frac{r-\rho}{\sigma}$. This the first best value for the growth of per capita commodity output. A comparison of this growth rate with that of the competitive economy (given in Equation [18]) reveals that in the competitive economy, a distortion appears in the growth rate due to imposition of the tax.

Given the value of $\phi_{command}$ and using the co-state equation and steady-state equilibrium conditions, we obtain unique solution of k from the following equation:

$$n = A(1-\phi_{command})^{\alpha} B^{1-\alpha} \phi_{command}^{(1-\beta)(1-\alpha)} k^{-\beta(1-\alpha)} [\{1-\beta(1-\alpha)\} - k\beta(1-\alpha)] \tag{39}$$

Let $h(k) = R.H.S$ of the aforementioned equation. Here, $h' < 0$. The existence of the unique value of k is shown diagrammatically in Figure 2. In Figure 3, the growth rate of population n and $h(k)$ is plotted on the vertical axis, and the value of k is plotted along the horizontal axis. From the figure, it is quite obvious that k has a unique value at steady state. In the same figure, $h(k)$ intersects the line of growth rate of population n, at E for equilibrium value of k. Let it be $\hat{k}$. Note that $\hat{k}$ does not depend on τ

In Figure 3, population growth is depicted by AB. CD represents function $h(k)$ from Equation (39). $\hat{k}$is the equilibrium value for k in command economy.

Given the equilibrium value of k, that is, $\hat{k}$, the value of optimal tax rate τ can be solved from the following equation which is obtained by using a steady-state equilibrium condition where the growth rates of human capital and consumption of commodity are equal to each other:

$$\hat{k}^{1-\beta(1-\alpha)} A(1-\phi_{command})^{\alpha} B^{1-\alpha} \phi_{command}^{(1-\beta)(1-\alpha)} \sigma\eta\tau = \{1-\beta(1-\alpha)\} \hat{k}^{-\beta(1-\alpha)} A(1-\phi_{command})^{\alpha} B^{1-\alpha} \phi_{command}^{(1-\beta)(1-\alpha)} - \rho \tag{40}$$

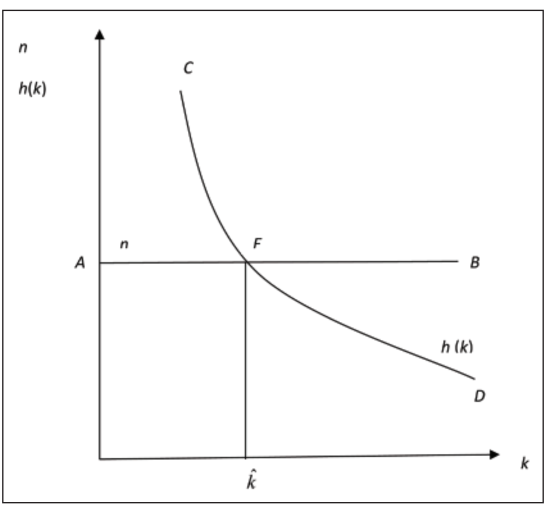

Figure 3. Determination of Optimal *k* in Command Economy

Source: The authors.

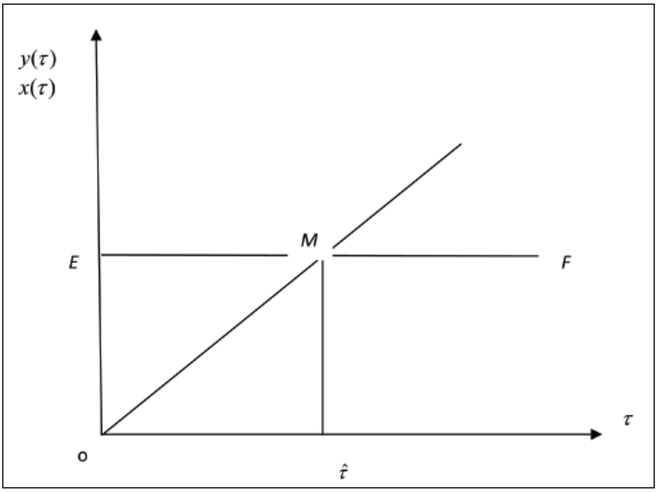

Figure 4. Determination of Optimal Tax rate in Command Economy

Source: The authors.

Let $x(\tau)$ be the LHS of Equation (40) and $y(\tau)$ be the RHS of Equation (40), where $x' > 0$ and $y' = 0$. In Figure 4, $x(\tau)$ intersects $y(\tau)$ at point M for unique τ.

In Figure 4, OH depicts the function $y(\tau)$ and EF represents $x(\tau)$ from Equation (40). $\hat{\tau}$ is the optimal tax rate in the command economy.

After obtaining the values of $\hat{k}$ and $\hat{\tau}$ at steady state, the values of growth rates can be derived. In steady state, the growth rate of consumption of commodity output and that of human capital accumulation are equal which is given by the following equation:

$$\gamma_h = \gamma_c = \left[\frac{\rho \hat{k} \eta \hat{\tau}}{\{1 - \beta(1-\alpha)\} - \hat{k}\eta\hat{\tau}\sigma} \right] \tag{41}$$

Proposition 4: *There exists positive, unique steady-state growth rate of human capital and that of consumption and production of commodity output in the command economy. There also exists a unique optimal tax rate.*

In the following section, we discuss another extension of the basic model.

Extension of the Basic Model

Household internalizes the government sector to itself and sets aside a part of their income for education.

In this section, we assume that there is no role of government, and household invests a fraction of their income in human capital education. In this case, all other equations remaining unchanged, the human capital and physical capital function are as follows:

$$\dot{h} = \frac{\eta\psi(whN + rK)}{N}$$

where ψ is the fraction of income invested for education; $0 < \psi < 1$

$$\dot{K} = \{(1-\psi)(whN + rK) - Nc\}$$

The Hamiltonian function can be formulated as:

$$H = \frac{c^{1-\sigma}}{(1-\sigma)}N + \theta_1[(1-\psi)(rK + wNh) - cN] + \theta_2[\eta\psi\frac{(whN + rK)}{N}] \tag{42}$$

Here, c, ψ are the decision variables and K and h are the state variables.[8]

We can derive the values of w, r, p_s using the profit maximization conditions like the basic model described in section 'Households, Firms and Government'. Using the no arbitrage condition, the values for φ and ψ are found as:

$$\left.\begin{aligned} \hat{\varphi} &= \frac{(1-\alpha)(1-\beta)}{1-\beta(1-\alpha)} \\ \hat{\psi} &= \frac{n-\rho+w\eta}{\eta\sigma(w+rk)} \end{aligned}\right\} \tag{43}$$

The growth rate of commodity sector is found as:

$$\gamma_c = \frac{n-\rho+w\eta}{\sigma} = \frac{r-\rho}{\sigma} \tag{44}$$

[8] Detailed derivation of this model is available with the author.

Thus, in case of private spending, we obtain the first best solution as equilibrium outcome in the absence of any distortion due to taxation.

$$r - w\eta = n \tag{44a}$$

where $r = \alpha A(1-\hat{\varphi})^{\alpha-1} B^{1-\alpha} \hat{\varphi}^{(1-\beta)(1-\alpha)} k^{-\beta(1-\alpha)}$

Replacing the values of w and r, we get Equation (45) that uniquely defines k in terms of the parameters.

$$A(1-\phi)^{\alpha}(1-\alpha)B^{1-\alpha}k^{-\beta(1-\alpha)}\varphi^{\alpha\beta-\alpha-\beta}\left[(1-\beta)-\eta\beta(\phi k)\right] = n \tag{45}$$

Let $\Gamma(k) = A(1-\phi)^{\alpha}(1-\alpha)B^{1-\alpha}k^{-\beta(1-\alpha)}\varphi^{\alpha\beta-\alpha-\beta}\left[(1-\beta)-\eta\beta(\phi k)\right]$

It can be shown that $\Gamma'(k) < 0$. Hence, there exists a unique k that satisfies Equation (45). Hence, using this, we find that

$$\frac{dk}{d\eta} = -\frac{[\beta(\phi k)]}{[\eta\beta\phi(1-\beta(1-\alpha)) + (1-\alpha)(1-\beta)\beta k^{-1}} < 0 \tag{46}$$

Thus, in the model with private spending too, we observe that k is negatively related to η. This result is the same as found in Equation (24), under competitive framework for basic model. Therefore, when the households are privately spending for accumulation of human capital, as the technological efficiency of education sector increases, per capita physical capital in steady state will decrease.

Using Equation (44), we observe that

$$\frac{d\gamma_c}{d\eta} = \frac{1}{\sigma}\frac{dr}{dk}\frac{dk}{d\eta} > 0 \quad \text{as } \frac{dr}{dk} < 0.$$

Hence, we conclude that in the model with producer services, when household spends a part of their income for accumulation of human capital, an increase in efficiency of education technology will cause the growth rate of per capita consumption to improve. This result is also similar to the result obtained in the basic model discussed in section ‘Basic Model’.

Numerical Analysis of Basic Model with Sector-specific Inputs

In this section, we develop a simpler version of the generalized model. Here, we assume that human capital is used only in service sector, and physical capital is employed only in the manufacturing sector. The general model is simplified by assuming $\beta = 1$.

The production functions are modified in the following manner:

$$Y_c = AK^{\alpha} Y_s^{1-\alpha} \tag{47}$$

$$Y_s = B(Nh) \tag{48}$$

Using the production functions, the human capital accumulation function is as follows:

$$\gamma_h = \frac{\dot{h}}{h} = \eta\left(\frac{G}{Nh}\right) = \eta\left(\frac{\tau y_c}{Nh}\right) = \eta\tau AB^{1-\alpha}k^{\alpha} \tag{49}$$

The investment function, utility function, balanced budget equation and growth path of population remain the same as the original model.

Given this assumption, section 'Competitive Economy' derives the steady-state growth path in competitive economy and section 'Command Economy' elaborates the same for a command economy. A comparative static analysis is done in section 'Comparative Static Results'.

Competitive Economy

In market economy, an individual consumer maximizes present discounted value of utility (over the infinite horizon) by choice of the consumption path subject to the wealth accumulation constraint. By using the current value Hamiltonian function, the problem of dynamic optimization is solved. The current value Hamiltonian for this particular problem is as follows:

$$H = \frac{N}{(1-\sigma)}(c^{1-\sigma} - 1) + \theta[(1-\tau)(rK + wNh) - cN]$$

In this problem, c is the decision variable, K is the one and only state variable and θ is the shadow price of physical capital. While solving this Hamiltonian function, tax rate τ is considered to be given throughout the analysis.

Steady-state Growth Path

The model results show that along the steady-state growth path, c, h and K grow at constant rate, and the growth rate of human capital and that of per unit commodity output consumption are given by

$$\gamma_h = \eta\tau AB^{1-\alpha}k^{\alpha} \tag{50}$$

$$\gamma_c = (1-\tau)\frac{A\alpha B^{1-\alpha}k^{\alpha-1}}{\sigma} - \frac{\rho}{\sigma} \tag{51}$$

The input prices, that is, r, w and p_s are found from the profit-maximizing conditions

$$r = A\alpha B^{1-\alpha}k^{\alpha-1}$$
$$w = p_s B$$
$$p_s = A(1-\alpha)B^{-\alpha}k^{\alpha}$$

In steady state, the growth rate of commodity output and that of human capital accumulation are equal, and equating these two expressions, we have

$$(1-\tau)\frac{A\alpha B^{1-\alpha}k^{\alpha-1}}{\sigma} - \frac{\rho}{\sigma} = \eta\tau AB^{1-\alpha}k^{\alpha} \tag{52}$$

As τ is given in competitive economy, the value of physical capital per skilled labour, that is, k is solved from the aforementioned equation.

Using Equation (35), we also find that

$$\frac{dk}{d\tau} = \frac{-[\alpha+\sigma\eta k]}{\alpha[\sigma\eta\tau + (1-\tau)(1-\alpha)k^{-1}]} < 0$$

To determine the growth-maximizing tax rate in competitive economy, the growth rate γ_c is maximized with respect to τ. The first-order condition is given as follows:

$$\frac{d\gamma_c}{d\tau} = 0$$

Using Equation (33), the level of tax rate that maximizes the growth rate in competitive framework is obtained. Let the level of tax rate be τ^{**}, and it is found that

$$\tau^{**} = (1 - \alpha) \tag{53}$$

The second-order condition that is required for growth maximization is $\frac{d^2\gamma_c}{d\tau^2} < 0$, and this condition is satisfied.

Proposition 5: *If human capital is the only input of service sector, then growth-maximizing tax rate in competitive economy is equal to the output elasticity of service in producing commodity output.*

In the present framework, in the competitive economy, the growth-maximizing tax rate is found to be the same as the output elasticity of manufacturing product with respect to service good. In this model, commodity sector uses service output as an input, which in turn is produced using human capital. Human capital accumulation depends on government per capita expenditure on education sector and financed by tax on commodity output. Hence, growth is maximized when tax rate is equal to the output elasticity of manufacturing product with respect to service good.

Command Economy

In the command economy, optimum solution is obtained by maximizing the following current value Hamiltonian

$$H = \frac{N}{(1-\sigma)}(c^{1-\sigma} - 1) + \theta_1 \dot{K} + \theta_2 \dot{h} \tag{54}$$

where θ_1 and θ_2 are shadow prices associated with $\dot{K}$ and $\dot{h}$ which stand for physical investment and human capital accumulation.

Here c and τ are the decision variables and K, h are state variables.

The growth rate of per capita commodity output is

$$\gamma_c = \frac{A\alpha k^{\alpha-1} B^{1-\alpha} - \rho}{\sigma} \tag{55}$$

The growth rate of human capital accumulation is

$$\gamma_h = \frac{\dot{h}}{h} = \eta\tau AB^{1-\alpha}k^{\alpha} \tag{56}$$

At steady state, $\gamma_h = \gamma_c$. Substituting the values, we get

$$\sigma\eta\tau AB^{1-\alpha}k^{\alpha} = A\alpha k^{\alpha-1}B^{1-\alpha} - \rho \tag{57}$$

Using the first-order conditions and the co-state equations, we get the following equation that solves k in terms of parameters.

$$n = AB^{1-\alpha}k^{\alpha-1}\alpha - Ak^{\alpha}B^{1-\alpha}(1-\alpha)\eta \tag{58}$$

Here, we get a non-linear solution of k. For obtaining the value of tax rate in command economy and to compare it with that of competitive economy which maximizes growth rate, the model is made simpler by assuming $\alpha = \frac{1}{2}$.

The value of k is

$$(\hat{\hat{k}}) = \frac{(\sqrt{n^2 + A^2B\eta} - n)^2}{A^2B\eta^2} \tag{59}$$

Substituting the value of $\alpha = \frac{1}{2}$ into Equation (58), the following equation is obtained:

$$n = AB^{\frac{1}{2}}(\hat{\hat{k}})^{-\frac{1}{2}}(\frac{1}{2}) - A(\hat{\hat{k}})^{\frac{1}{2}}B^{\frac{1}{2}}(\frac{1}{2})\eta \tag{58$'$}$$

Substituting the equilibrium value of k, that is, $(\hat{\hat{k}})$ in Equation (57), the value of optimal tax rate is solved. The optimal tax rate is

$$\tau_{command} = \frac{A(\frac{1}{2})(\hat{\hat{k}})^{-\frac{1}{2}}B^{\frac{1}{2}} - \rho}{\eta AB^{\frac{1}{2}}(\hat{\hat{k}})^{\frac{1}{2}}\sigma} = \frac{\gamma_c}{\eta AB^{\frac{1}{2}}(\hat{\hat{k}})^{\frac{1}{2}}} \tag{60}$$

Now the value of tax rate must be positive, and its value must be less than the one which implies $0 \le \tau_{command} \le 1$. From Equation (44), it is clear that $\tau_{command} \ge 0$ if γ_c is positive as all the other variables in the expression are assumed to be positive by default.

Therefore, the required condition for positive value of tax rate is

$$\gamma_c \geq 0 \tag{61}$$

Substituting the value of $\alpha = \frac{1}{2}$ into the commodity production function, it is found that $A(\frac{1}{2})(\hat{\hat{k}})^{-\frac{1}{2}} B^{\frac{1}{2}}$ is the expression of marginal productivity of physical capital.

The aforementioned condition implies that the value of marginal productivity of capital should be higher than the rate of time preference for tax rate to be positive. It is because the use of human capital is solely to increase future production. If individuals discount future more heavily compared to present marginal productivity of capital, it is meaningless to invest in human capital.

For $\tau_{command} \leq 1$, the required condition is

$$A(\frac{1}{2})(\hat{\hat{k}})^{-\frac{1}{2}} B^{\frac{1}{2}} \leq \rho + \eta A B^{\frac{1}{2}} (\hat{\hat{k}})^{\frac{1}{2}} \sigma$$

Using Equation (58′), we obtain

$$n - \rho \leq A B^{\frac{1}{2}} (\hat{\hat{k}})^{\frac{1}{2}} \eta (\sigma - \frac{1}{2}) \tag{62}$$

or

$$n - \rho \leq (\sqrt{n^2 + A^2 B \eta} - n)(\sigma - \frac{1}{2})$$

Therefore, for $\tau_{command} \leq 1$
or

$$\frac{n - \rho}{(\sqrt{n^2 + A^2 B \eta} - n)} + \frac{1}{2} \leq \sigma \tag{63}$$

Thus, in general model, the steady-state growth rates of human capital, commodity consumption and that of physical capital are obtained endogenously under both competitive and command economic regimes. Considering the tax rate as given in competitive frame, the values of k, ϕ are determined. In command economy, it is clear from the model that optimal tax rate exists. In the sector-specific input model along with the uniquely determined steady-state growth rates for human capital, physical capital and commodity consumption, the value of growth-maximizing tax rate under competitive regime and the optimal tax rate under command economy are also derived. The value of growth-maximizing tax rate in competitive economy is equal to the value of output elasticity of service in producing final output (α), and its value is assumed to be constant for the model under consideration. However, it is obvious from the expression of optimal tax rate (derived in earlier section) that the value of it depends on different parameters. In the following section, the comparative static analysis is done to study the impact of different parameters on optimal tax rate obtained in command economy framework.

Comparative Static Results

Differentiating optimal tax rate in command economy in this special case with respect to rate of time preference, we find

$$\frac{d\tau_{command}}{d\rho} = -\frac{1}{\eta AB^{1-\alpha}(k)^{\alpha}\sigma} < 0 \tag{64}$$

As rate of time preference (discount rate) rises, individuals become more concerned for present consumption rather than future consumption. So, under balanced budget, tax rate that raises the future accumulation rate of human capital which will be used as an input to produce commodity output in subsequent periods will fall for a rise in rate of time preference.

Now, we study how the optimal tax rate will respond when the technology parameter in human capital accumulation changes. Differentiating optimal tax rate with respect to technology parameter of human capital accumulation, we find

$$\frac{d\tau_{command}}{d\eta} = \frac{-\frac{dk}{d\eta}\alpha[(1-\alpha)+\sigma\eta k]}{\sigma k^{2}\eta} - \frac{\tau}{\eta} \tag{65}$$

where

$$\frac{dk}{d\eta} = \frac{-k^{\alpha}}{(\alpha k^{\alpha-2}+\eta\alpha k^{\alpha-1})} < 0 \tag{66}$$

In numerical simulation done in the next section, we find $\frac{d\tau_{command}}{d\eta} > 0.$

This result is intuitively obvious. As efficiency of education sector η rises, marginal benefit from investing one unit for accumulation of human capital increases. Hence, it is optimal to increase tax rate.

Differentiating optimal tax rate with respect to the inverse of inter-temporal elasticity of substitution, we find that

$$\frac{d\tau_{command}}{d\sigma} = -\frac{\tau_{command}}{\sigma} < 0 \tag{67}$$

where σ stands for the inverse of inter-temporal elasticity of substitution. As inter-temporal elasticity of substitution increases, representative consumer can easily substitute present consumption by future consumption, and optimal tax rate increases because tax can finance education that can generate human capital in future. That human capital can be used for service production, and service is again used for commodity output production in future. So when σ decreases, people are ready to forgo present consumption for future consumption and willing to pay more tax.

Proposition 6: *The optimal tax rate decreases as the discount rate (ρ) increases and/or the elasticity of marginal utility and inverse of which is known as inter-temporal elasticity of substitution (σ)increases. The optimal tax rate increases as efficiency parameter of human capital accumulation (η) increases.*

Numerical Example

This section gives a numerical example to substantiate the results obtained in section 'Extension of the Basic Model'. Here, we consider a special case of specific factor model with $\alpha = 1/2$.

Under $\alpha = \frac{1}{2}$ assumption, the ratio of physical capital to human capital, growth-maximizing tax rate, growth rate of commodity output and growth rate of human capital under competitive economy are as follows:

$$k_{competitive} = \left[\frac{-\rho + \sqrt{\rho^2 + 4(.125)A^2 B\sigma\eta}}{\sigma\eta A B^{\frac{1}{2}}}\right]^2$$

$$\tau_{competitive} = (1 - \alpha) = (1 - .5) = .5$$

$$\gamma_c^{competitive} = \frac{A(.25)B^{\frac{1}{2}}k^{-\frac{1}{2}}_{competitive}}{\sigma} - \frac{\rho}{\sigma}\gamma_h^{competitive} = \eta(.5)AB^{\frac{1}{2}}k^{\frac{1}{2}}_{competitive}$$

Under $\alpha = \frac{1}{2}$ assumption, the ratio of physical capital to human capital, growth-maximizing tax rate, growth rate of commodity output and growth rate of human capital obtained under command economic regime are as follows:

$$k_{Command} = \frac{(\sqrt{n^2 + A^2 B\eta} - n)^2}{A^2 B\eta}$$

$$\tau_{command} = \frac{A(.5)k^{-\frac{1}{2}}_{command}B^{\frac{1}{2}} - \rho}{\eta AB^{\frac{1}{2}}k^{\frac{1}{2}}_{command}\sigma}$$

$$\gamma_c = \frac{A(.5)k^{-\frac{1}{2}}_{command}B^{\frac{1}{2}} - \rho}{\sigma}$$

$$\gamma_h = \frac{\dot{h}}{h} = \eta\tau_{command}AB^{\frac{1}{2}}k^{\frac{1}{2}}_{command}$$

A few diagrammatic representations have been shown on the basis of the numerical analysis.

Numerically, it has been found that for $n = 0.01$, $A = 0.5$, $B = 0.5$, $\eta = 0.05$, $\sigma = 2.1$ and $0.05 \leq \rho \leq 0.1$ $\frac{d\tau_{command}}{d\rho} < 0$ which is illustrated in Figure 5. In this figure, we see that the optimal tax rate in command economy falls with the increase in values of rate of preference.

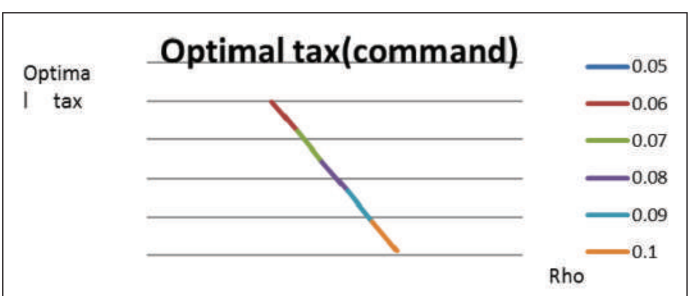

Figure 5. Relationship Between Optimal Tax Rate and ρ

Source: The authors.

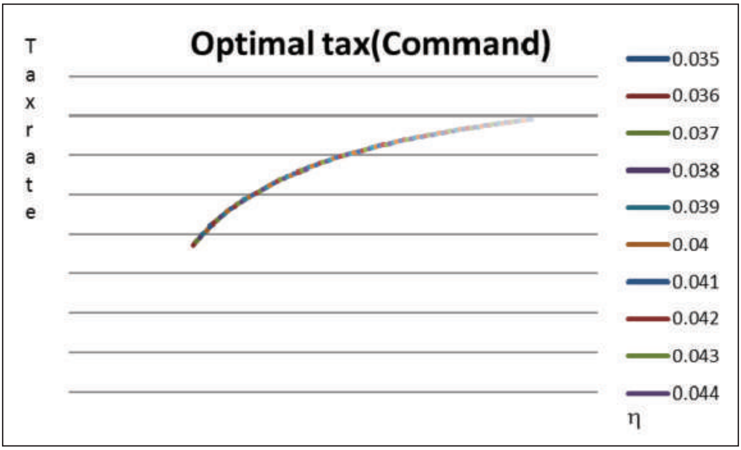

Figure 6. Relationship Between Optimal Tax Rate and η

Source: The authors.

Second, numerically, it is found that for $n = 0.01$, $A = 0.5$, $B = 0.5$, $\rho = 0.07$, $\sigma = 2.5$ and for the range $0.035 \leq \eta \leq 0.131$, the value of tax rate increases as η, that is, $\frac{d\tau_{command}}{d\eta} \geq 0$. The result is shown in Figure 6.

Finally, it is observed that for $n = 0.01$, $A = 0.5$, $B = 0.5$, $\rho = 0.07$, $\eta = 0.05$ and $1.5 \leq \sigma \leq 2.46$, $\frac{d\tau_{command}}{d\sigma} < 0$, that is, the value of tax rate decreases as σ rises (see Figure 7).

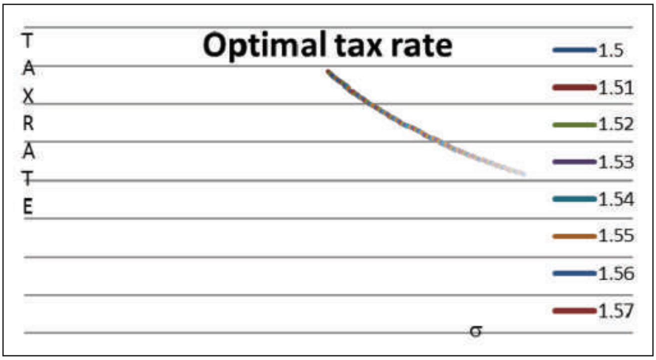

Figure 7. Relationship Between Optimal Tax Rate and σ

Source: The authors.

The value of the growth-maximizing tax rate under competitive economy rate is constant, that is, 0.5 here. Theoretically, it is derived that under certain assumptions, its value does not depend on any of the parameters of η, σ and ρ. Therefore, in every case, in earlier figures, the growth-maximizing tax rate will be a horizontal line with respect to the changing values of any parameter under consideration. The growth-maximizing tax rate in competitive economy is constant and depending on variation of different parameters, such as η, σ and ρ, it may be higher than the optimal tax rate of the command economy or the other way round.

Finally, we compare the growth rates for command and competitive economies and try to find how they vary with different parameters. In the following two figures, the growth rates of human capital under two economic regimes are plotted along the vertical axis, and the values of the parameter under consideration are plotted along the horizontal axis. The dotted line represents the growth rate of command economy, whereas the solid one denotes the growth rate of competitive economy.

Figure 8 shows the pattern of growth path of human capital in two different economic regimes due to change in value of technology parameter of human capital, that is, η. It is found that the growth rate in command economy is higher than that of the competitive economy.

In Figure 9, comparison is done between growth rates under command and competitive framework with variation in rate of time preference. The growth rate in command economy is higher than that of the competitive economy.

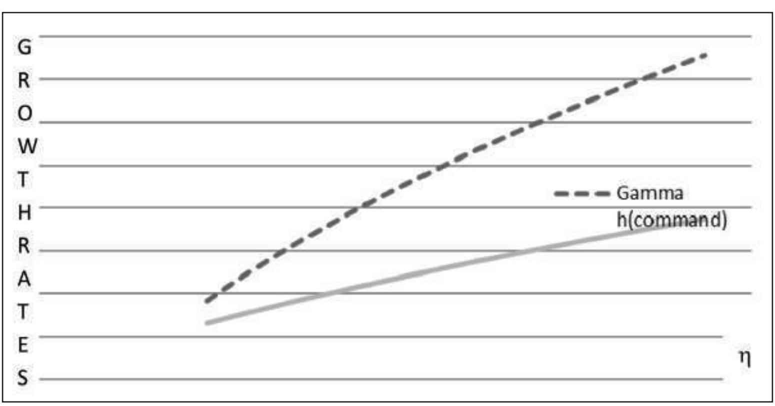

Figure 8. Comparison Between Growth Rates with Respect to η

Source: The authors.

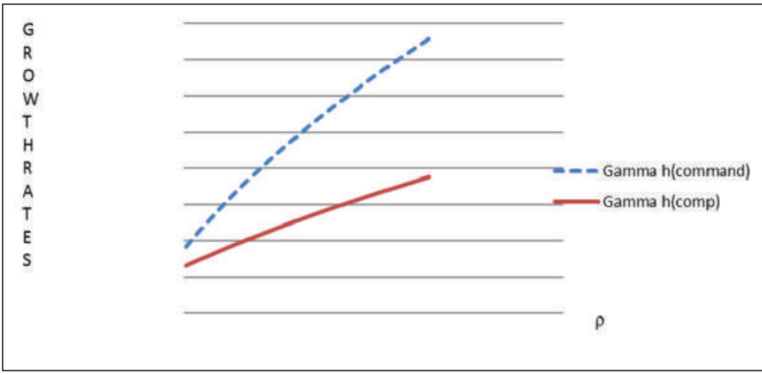

Figure 9. Comparison Between Growth Rates with Respect to ρ

Source: The authors.

Thus, from Figure 9, it is observed that the command economy has higher growth rate than that of competitive economy under this numerical specification. This result is found because in command economy, tax is chosen by the social planner optimally, whereas in decentralized economy, tax rate is considered to be exogenously given by the optimizing agents. Then, the growth rate in competitive economy is maximized with respect to tax rate. After substituting this growth by maximizing tax rate, competitive economy growth rate is obtained. Under command economic regime, the cost of taxation is being equalized with the benefit of taxation by the social planner at the margin. In competitive economy, the decisions taken are disjoint. Hence, follows the above result.

Conclusion

In this article, an endogenous growth model is considered with producer service. The service output is used as an intermediate good in commodity sector. Human capital is used to produce service good. Initially, we assume that accumulation of human capital depends on the government expenditure on education sector. The government levies tax on the commodity output. This model is considered as the basic model in the article. In this framework, we observe that there exists a unique saddle path stable steady-state growth rate of human capital accumulation, which works as the source of growth for all other sectors of the economy. Also, we observe in the competitive framework, a unique growth-maximizing tax rate exists. We also compare the optimal tax rates for the command economy with growth-maximizing tax rate in competitive economy under the assumption that the physical capital is specific to commodity manufacturing sector, and human capital is specific to producer service sector. It is found that the growth-maximizing tax rate in competitive economy is constant, and depending on the variation of different parameters, it may be higher than the optimal tax rate of the command economy or the other way around. Further, the tax rate of the command economy increases when the efficiency of human capital accumulation technology rises. However, the optimal tax rate decreases as the discount rate of utility and the elasticity of marginal utility increase. Finally, the numerical analysis shows that in the presence of producer service, the command economy will have a higher growth rate than the competitive economy even after imposition of growth-maximizing tax rate in competitive economy. We also attempt another extension of the model where it is assumed that households privately spend for accumulation of the human capital. In this case, we obtain the first best solution as the equilibrium rate of growth in the absence of distortion due to taxation. This same growth rate is also yielded by command economy after endogenizing the tax rate. In addition, like the basic model as the technological efficiency of the education sector improves, the growth of per capita consumption will improve.

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Appendix A

To Prove

$$rK + whN = Y_c$$

From Equations (13b), (14) and (15), substituting the value of r, w and p_s in the LHS of the above equation, we get

$$p_s B(Nh)^{\beta-1}\beta(\phi K)^{1-\beta}hN + p_s B(Nh)^{\beta}(1-\beta)(\phi K)^{-\beta}K$$

$$= p_s B(Nh)^{\beta}K^{1-\beta}\phi^{-\beta}[\beta\phi + (1-\beta)]$$

$$= p_s B(Nh)^{\beta}K^{1-\beta}\phi^{-\beta}[B\frac{(1-\alpha)(1-\beta)}{1-\beta(1-\alpha)} + (1-\beta)]$$

$$= p_s B(Nh)^{\beta}K^{1-\beta}\phi^{-\beta}[\frac{(1-\beta)}{1-\beta(1-\alpha)}]$$

$$= p_s \frac{(1-\beta)}{1-\beta(1-\alpha)}B(Nh)^{\beta}K^{1-\beta}\phi^{-\beta}$$

$$= [A(1-\phi)^{\alpha}(1-\alpha)B^{-\alpha}k^{\alpha\beta}\phi^{-\alpha(1-\beta)}]\{\frac{(1-\beta)}{1-\beta(1-\alpha)}B(Nh)^{\beta}K^{1-\beta}\phi^{-\beta}\}$$

$$= \frac{AB(Nh)^{\beta(1-\alpha)}(1-\phi)^{\alpha-1}B^{-\alpha}\phi^{(1-\alpha)(1-\beta)}K^{\alpha+(1-\alpha)(1-\beta)}\alpha}{\{1-\beta(1-\alpha)\}}$$

$$= AB^{1-\alpha}K^{\alpha+(1-\alpha)(1-\beta)}\phi^{(1-\alpha)(1-\beta)}(Nh)^{\beta(1-\alpha)}\frac{(1-\phi)^{\alpha}}{(1-\phi)\{1-\beta(1-\alpha)\}}$$

Now

$$1-\phi = 1-\frac{(1-\alpha)(1-\beta)}{1-\beta(1-\alpha)} = \frac{1-\beta(1-\alpha)-(1-\alpha)(1-\beta)}{1-\beta(1-\alpha)}$$

Therefore,

$$= AB^{1-\alpha}K^{\alpha+(1-\alpha)(1-\beta)}\phi^{(1-\alpha)(1-\beta)}(Nh)^{\beta(1-\alpha)}\frac{(1-\phi)^{\alpha}}{(1-\phi)\{1-\beta(1-\alpha)\}} =$$

$$\frac{Y_c}{1-\beta(1-\alpha)-(1-\alpha)+\beta(1-\alpha)} = Y_c(\text{proved})$$

References

Banga, R., & Goldar, B. (2007). Contribution of services to output growth and productivity in Indian manufacturing: Pre- and post-reforms. *Economic and Political Weekly*, *42*(26), 2769–2777.

Barua, A., & Pant, M. (2014). Trade and wage inequality: A specific factor model with intermediate goods. *International Review of Economics and Finance, 33*(C), 172–185.

Beauchemin, K. R. (2001). Growth or stagnation? The role of public education. *Journal of Development Economics*, *64*(2), 389–416.

Behuria, S., & Khullar, R. (1994). *Intermediate services and economic development: The Malaysian example*. Retrieved from https://www.adb.org/publications/intermediate-services-and-economic-development-malaysian-example

Blankenau, W. F., Simpson, N. B., & Tomljanovich, M. (2007). Public education expenditures, taxation, and growth: Linking data to theory. *American Economic Review*, *97*(2), 393–397.

Bosworth, B., Collins, S. M., & Virmani, A. (2006). Sources of growth in the Indian economy. *India Policy Forum*, *3*(1), 1–69. Retrieved from https://www.brookings.edu/wp-content/uploads/2016/07/2006_bosworth.pdf

Cardak, B. A. (2004). Education choice, endogenous growth and income distribution. *Economica*, *71*(281), 57–81.

Chen, B. L., & Lee, S. F. (2007). Congestible public goods and local indeterminacy: A two-sector endogenous growth model. *Journal of Economic Dynamics and Control*, *31*(7), 2486–2518.

Das, S. P., & Saha, A. (2015). Growth of business services: A supply-side hypothesis. *Canadian Journal of Economics/Revue canadienned'économique*, *48*(1), 83–109. doi:10.1111/caje.12120

Dasgupta, D. (1999). Growth versus welfare in a model of nonrival infrastructure. *Journal of Development Economics*, *58*(2), 359–385.

De la Fuente, A., & Doménech, R. (2006). Human capital in growth regressions: How much difference does data quality make? *Journal of the European Economic Association*, *4*(1), 1–36.

Faig, M. (1995). A simple economy with human capital: Transitional dynamics, technology shocks, and fiscal policies. *Journal of Macroeconomics*, *17*(3), 421–446.

Fernández, E., Novales, A., & Ruiz, J. (2004). Indeterminacy under non-separability of public consumption and leisure in the utility function. *Economic Modelling*, *21*(3), 409–428.

Francois, J., & Woerz, J. (2008). Producer services, manufacturing linkages, and trade. *Journal of Industry, Competition and Trade*, *8*(3–4), 199–229.

Futagami, K., Morita, Y., & Shibata, A. (1993). Dynamic analysis of an endogenous growth model with public capital. *The Scandinavian Journal of Economics*, *95*(4), 607–625.

Garcia-Castrillo, P., & Sanso, M. (2000). Human capital and optimal policy in a Lucas-type model. *Review of Economic Dynamics*, *3*(4), 757–770.

Glomm, G., & Ravikumar, B. (2001). Human capital accumulation and endogenous public expenditures. *Canadian Journal of Economics/Revue canadienned'économique*, *34*(3), 807–826.
Gomez, M. A. (2003). Optimal fiscal policy in the Uzawa-Lucas model with externalities. *Economic Theory*, *22*(4), 917–925.
Greiner, A. (2006). Human capital formation, public debt and economic growth. *Journal of Macroeconomics*, *30*(1), 415–427.
Hollanders, H. J. G. M., & ter Weel, B. J. (1999). Skill-biased technical change: On endogenous growth, wage inequality and government intervention. In H. Hagemann & S. Seiter (Eds.), *Growth theory and growth* policy (pp. 156–171). London, UK: Routledge.
Imbruno, M. (2014). *Trade liberalization, intermediate inputs and firm efficiency: Direct versus indirect modes of import*. Nottingham Centre for Research on Globalisation and Economic Policy, University of Nottingham, Nottingham, UK.
Ishikawa, J. (1992). Learning by doing, changes in industrial structure and trade patterns, and economic growth in a small open economy. *Journal of International Economics*, *33*(3–4), 221–244.
Lucas, R. E., Jr. (1988). On the mechanics of economic development. *Journal of Monetary Economics*, *22*(1), 3–42.
Mankiw, N. G., Romer, D., & Weil, D. N. (1992). A contribution to the empirics of economic growth. *The Quarterly Journal of Economics*, *107*(2), 407–437.
Miroudot, S., Lanz, R., & Ragoussis, A. (2009). *Trade in intermediate goods and services* (OECD Trade Policy Working Paper No. 93). Paris: OECD.
Moro, A. (2007). *Intermediate goods and total factor productivity* (Economics Working Papers we076034). Universidad Carlos III de Madrid. Departamento de Economía. Madrid, Spain.
Psarianos, I. N. (2002). Fiscal policy in an endogenous growth model with horizontally differentiated intermediate goods. *Spoudai*, *52*(4), 18–41.
Riley, G. (2014). *Economic growth-the role of human & social capital, competition & innovation*. Retrieved from http://www.tutor2u.net/economics/revisionnotes/a2-macro-economic-growth-capital.html
Sekar, M., Delgado, D. R., & Ulu, M. F. (2015). *Imported intermediate goods and product innovation: Evidence from India* (Working Papers 1537). Research and Monetary Policy Department, Central Bank of the Republic of Turkey, Turkey.
Tanaka, R. (2003). Inequality as a determinant of child labour. *Economics Letters*, *80*(1), 93–97.
Tsoukis, C., & Miller, N. J. (2003). Public services and endogenous growth. *Journal of Policy Modeling*, *25*(3), 297–307.
Woo, J. (2005). Social polarization, fiscal instability and growth. *European Economic Review*, *49*(6), 1451–1477.