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Endogenous Growth Model In A Two Sector Competitive Economy: Service Good Used As A Factor Input In Commodity Sector

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Abstract: This paper considers a closed economy model with two sectors namely, commodity sector and service sector. The service output is exclusively used as an intermediate input in producing commodity output. Here, in this model, an endogenous growth model is considered in which the service output is used as an intermediate good in commodity sector. Accumulation of human capital depends on the government expenditure on education sector. The government levies tax on the commodity output. In this framework, on the basis of the unique steady state growth path, where human capital accumulation works as the source of growth for all other sectors of the economy, a comparative static analysis has been done. In this particular model, we have analysed, how the share of physical capital that is engaged to the service sector is influenced by the commodity output elasticity of physical capital and service output elasticity of skilled labour.

Section 1: Introduction

The last few decades have experienced a rapid growth in the service sector, which has been reflected in an upward trend in the usage of service goods as consumption good, as well as intermediate good. There exists a substantial empirical literature dealing with the issues related to service sector. However, there are very few papers that consider service output as an intermediate good in an endogenous growth model and none of these studies considers any comparative analysis of taxation in command and market economy framework. In this model also, we assume human capital is one of the most important factor in producing service. Most of these services such as education, health, public administration, banks, computer services, recreation, communications, financial services and many others require specialized knowhow. The role of government expenditure on public resources is, to create both, human capital and physical capital, through taxation in endogenous growth models.

The aim of the present paper is to figure out a comparative static analysis on the basis of the optimal growth path in a competitive economy where service sector output is used as an input in the manufacturing sector.

There exists huge literature on intermediate goods. Few of them are mentioned here in below. The papers by **Moro (2007), Sekar et al (2015), Barua and Pant (2013), Imbruno(2012)** have discussed the matters connected to various aspects of intermediate goods in the context of closed or open economies. Seker et al (2015) have tried to develop a general equilibrium model of multi-product firms that explain the relationship between importing intermediate goods, innovation, and firm growth. This paper assumes heterogeneity in firms' efficiency levels as per the model built by Melitz (2003) and following the structural models by **Klette and Kortum (2004) and Seker (2009)**, a stochastic dynamic model of firm on industry evolution is formed. In the paper by Barua and Pant (2013), a theoretical model is built to explain the phenomenon of increasing relative wage inequality between unskilled and skilled wages, in a general equilibrium framework by incorporating intermediate goods. **Imbruno (2011)** attempts to study the impact of trade liberalization in intermediate inputs within a general equilibrium framework built by Melitz (2003), where all firms are assumed to be heterogeneous in productivity and can produce either intermediate goods or final goods under monopolistic competition.

Psarianos I. N (2002) has built an endogenous growth theory and it has been found theoretically and empirically that market economies invest very nominal amount of money in scientific and technical research and the growth rates are found to be lower than socially optimal.
Given the various models on intermediate good, we observe none except **Ishikawa (1991)** has captured service as an intermediate good in the endogenous growth.
The paper by Ishikawa (1991) has considered service as an input to produce manufacturing commodity and in this paper learning by doing in the service sector is the engine of endogenous growth. Following Ishikawa (1991) the present model assumes that service output is used as an input in the manufacturing sector that produces a malleable good used both for consumption and investment in a closed economy. Unlike Ishikawa (1991) we assume that service sector uses human capital, which is accumulated through government expenditure on education sector. Government expenditure is financed by imposing tax on manufacturing sector. We find out growth maximizing tax rate in competitive economy, where the tax is given. One of my papers **written by me and my co-authors,** Gupta et. al (2019), titled "Service good as an intermediate input and optimal government policy in an endogenous growth-model"**,** has left a few comparative static analysis. I have tried to figure out those undone analysis.
The rest of the paper is organised as follows: In section 2, the basic model is presented; In Section 3, a simplified version of the model is built where only human capital is used in service sector. The growth rates are determined in competitive economy along with the comparative static analysis and section 4 concludes.

2. The model:

This section, discusses the basic assumptions of the model and derives the growth path under competitive framework.

Section 2.1: The Households, Firms and Government

This paper considers a closed economy model with two sectors namely, commodity sector and service sector. The service output is exclusively used as an intermediate input in producing commodity output. The total labour force is homogeneous as far as skill is concerned. Identical rational agents inhabit the economy. Production technology is subject to constant returns to scale. The household sector chooses the path of per capita consumption of commodity output by maximising the integral over all future time of discounted instantaneous utility, ρ being the discount rate and σ, the elasticity of marginal utility and inverse of which is known as inter temporal elasticity of substitution. N represents the total labour force or working population.
Preferences over consumption are given by the following function where 'c' denotes flow of real per capita consumption of commodity output:

$$u(c) = \int_0^{\infty} \frac{(c^{1-\sigma} - 1)}{(1-\sigma)} e^{-\rho t} N(t)dt \qquad 1.$$

Here, we assume that the output in the commodity sector can be used for consumption or investment purposes, whereas the output in the service sector is used as a prime factor to produce commodity output.

The commodity output is produced using physical capital and service product. The service output is produced with human capital and physical capital. Both the production functions are Cobb-Douglas type. Here 'K' stands for the level of physical capital. Let α and β be the commodity output elasticity of physical capital and service output elasticity of skilled labour respectively. The commodity and service output production functions can be written as

$$y_c = A\{(1-\varphi)K\}^{\alpha} \, y_s^{\,1-\alpha} \qquad 2.$$

$$y_s = B(Nh)^{\beta} (\varphi K)^{1-\beta} \qquad 3.$$

Where 'y_c' and 'y_s' are the flow of commodity output and service output respectively. It is obvious that $(1-\alpha)$ measures the commodity output elasticity of service product. Similarly, $(1-\beta)$ measures the service output elasticity of physical capital. The level of population is growing at an exponential rate in the following manner:

$$N(t) = N_0 e^{nt} \qquad 4.$$

Where N_0 is the population size at initial time period. It is assumed that the initial amount of population $N_0 = 1$. Further, we assume that the general skill level of a worker is 'h'. The effective skilled labor input in commodity production is 'Nh'. Let 'φ' be the fraction of physical capital that is dedicated to the service sector. The remaining (1- φ) is engaged in producing commodity output.

It is assumed that Government spends money on working population to create human capital.

The human capital accumulation can be written as

$$\dot{h} = \eta \frac{G}{N} \qquad 5.$$

Here η be the technology parameter of human capital accumulation and G be the government expenditure on education. It is further assumed that only the commodity sector is being taxed. The tax revenue is spent as government expenditure to build human capital. Let the tax rate be τ which is levied on per unit of commodity output. The balanced budget equation can be written

$$G = T = \tau y_c \qquad 6.$$

A part of disposable income is consumed and the rest is invested to form physical capital. Hence, the physical capital accumulation function is given by

$$\dot{K} = (1-\tau)y_c - Nc \qquad 7.$$

In market economy, the households own all capital. The final product is produced by a representative firm that maximises profit. The objective of the economy is to maximize the present discounted value of utility over the infinite time horizon defined by equation (1) subject to the wealth accumulation constraint.

Section 3: Competitive Economy: The comparative static analysis

The objective of an individual consumer is to maximize utility choosing the consumption path. The current value Hamiltonian for this particular problem follows as

$$H = \frac{c^{1-\sigma}-1}{(1-\sigma)} N(t) + \theta[(1-\tau)(rK + wNh) - cN] \qquad 8.$$

In competitive economy, a representative household chooses c, the flow of consumption. So, c is the decision variable and K is state variable, θ is the shadow price of physical capital. While solving this Hamiltonian function, tax rate τ is considered to be given as per the competitive regime.

The profit of the producers for the commodity sector and service are

$$\pi_c = p_c y_c - r(1-\varphi)K - p_s y_s \qquad 9.$$

$$\pi_s = p_s y_s - (r\varphi K) - (wNh) \qquad 10.$$

Here r is the rate of interest and w is the real wage rate, p_s is the per unit price of service output. It is assumed that the commodity output is numeraire commodity which implies that per unit price of commodity output, i.e., p_c is unity.

The output and the factor markets both are characterized by perfect competition. Hence, equating the value of the marginal product of each factor input to its return and using profit maximisation condition we get the following expressions of r, w and p_s

$$r = A\alpha(1-\varphi)^{\alpha-1}B^{1-\alpha}k^{-\beta(1-\alpha)}\varphi^{(1-\beta)(1-\alpha)} \qquad 11.$$

$$w = Bp_s\beta\varphi^{1-\beta}k^{1-\beta} \qquad 12.$$

$$p_s = A(1-\varphi)^{\alpha}(1-\alpha)B^{-\alpha}k^{\alpha\beta}(\varphi)^{-\alpha(1-\beta)} \qquad 13.$$

Here k is defined as the physical capital per unit of skilled labour, i.e., $k = \frac{K}{hN}$. The value of φ is solved from the above system of equations.

The value of φ is

$$\hat{\varphi} = \frac{(1-\alpha)(1-\beta)}{1-\beta(1-\alpha)} \qquad 14.$$

The steady state growth paths in market economy is defined as the path along which c, h, K grow at constant rate and the value of φ is time independent. The growth rate of human capital accumulation and that of per unit commodity output consumption and the growth rate of physical capital are given by

$$\gamma_h^{comp} = \eta\tau A\{(1-\hat{\varphi})\}^{\alpha}B^{1-\alpha}(\hat{\varphi})^{(1-\beta)(1-\alpha)}k^{1-\beta(1-\alpha)} \qquad 15.$$

$$\gamma_c^{comp} = \frac{(1-\tau)\alpha A(1-\hat{\varphi})^{\alpha}B^{1-\alpha}\hat{\varphi}^{(1-\beta)(1-\alpha)}k^{-\beta(1-\alpha)} - \rho}{\sigma} \qquad 16.$$

$$\gamma_K^{comp} = n + \gamma_h^{comp} \qquad 17.$$

Where γ_x stands for growth rate of the variable x.

Along the steady state, $\gamma_c = \gamma_h$ (See equation (A.28) in Appendix for details). Equating γ_c, γ_h from equation (15) and (16), the following equation is obtained in terms of k, τ and other parameters. τ is considered to be given in competitive economy.

From the steady state $\gamma_c = \gamma_h$ equation, it is can be shown that there exists unique value of k in terms of τ, which is given in competitive economy.

Therefore, there exists positive, unique steady state growth rate for human capital, physical capital and production and consumption of commodity output in competitive economy.

Comparative Static Analysis:

We have done a comparative static analysis on the fraction of physical labor that is devoted to the service sector.

The value of the share of the physical capital

$$\hat{\varphi} = \frac{(1-\alpha)(1-\beta)}{1-\beta(1-\alpha)}$$

In this model, 'φ' be the fraction of physical capital that is dedicated to the service sector. In this model, α and β are the commodity output elasticity of physical capital and service output elasticity of skilled labour respectively.

Differentiating φ with respect to α we get

$$\frac{\partial\varphi}{\partial\alpha} = [\frac{-\{1-\beta(1-\alpha)\}(1-\beta)-(1-\alpha)(1-\beta)\beta}{\{1-\beta(1-\alpha)\}^2}]\beta = \frac{\beta(1-\beta)}{\{1-\beta(1-\alpha)\}^2}$$

Or,

$$\frac{\partial\varphi}{\partial\alpha} = \frac{\beta(1-\beta)}{\{1-\beta(1-\alpha)\}^2} \qquad [$$

Proposition 1: The fraction of physical capital that is dedicated to the service sector increases, when the commodity output elasticity of physical capital rises.

The logic behind the proposition is, when the relative responsiveness of commodity output to the physical capital is positive, the more amount of physical capital will be required. The investment on physical capital will be required by the economy to maintain positive growth.

Differentiating φ with respect to β

$$\frac{\partial\varphi}{\partial\beta} = [\frac{-\{1-\beta(1-\alpha)\}\{(1-\alpha)(-1)-(1-\beta)(1-\alpha)\{-(1-\alpha)\}}{\{1-\beta(1-\alpha)\}^2}][-(1-\alpha)]$$

Or, $\frac{\partial\varphi}{\partial\beta} = [\frac{-\{1-\beta(1-\alpha)\}\{(1-\alpha)(-1)-(1-\beta)(1-\alpha)\{-(1-\alpha)\}}{\{1-\beta(1-\alpha)\}^2}][-(1-\alpha)]$

Or, $\frac{\partial\varphi}{\partial\beta} = \frac{\{1-\beta(1-\alpha)\}\frac{\partial}{\partial\beta}[(1-\alpha)(1-\beta)]-[(1-\alpha)(1-\beta)]\frac{\partial}{\partial\beta}\{1-\beta(1-\alpha)\}}{\{1-\beta(1-\alpha)\}^2}$

Or, $\frac{\partial\varphi}{\partial\beta} = \frac{\{1-\beta(1-\alpha)\}(1-\alpha)(-1)-(1-\alpha)(1-\beta)\{-(1-\alpha)\}}{\{1-\beta(1-\alpha)\}^2}[-(1-\alpha)]$

Or, $\frac{\partial\varphi}{\partial\beta} = \frac{\alpha(1-\alpha)^2}{\{1-\beta(1-\alpha)\}^2}$

Proposition 2: The fraction of physical capital that is dedicated to the service sector increases, if the service output elasticity of skilled labour rises.

The reason behind such result is, when the relative responsiveness of service output to the change in skilled labour increases, the share of physical capital allotted to service sector has to increase, to maintain the capital - labour ratio for the growth of service output production.

Conclusion:

In this paper an endogenous growth model is considered in which the service output is used as an intermediate good in commodity sector. Human capital is used to produce service good. Accumulation of human capital depends on the government expenditure on education sector. The government levies tax on the commodity output. In this framework, on the basis of the unique steady state growth path, where human capital accumulation works as the source of growth for all other sectors of the economy, a comparative static analysis has been done. In this particular model, we have analysed, how the share of physical capital that is dedicated to the service sector is influenced by the commodity output elasticity of physical capital and service output elasticity of skilled labour.

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Appendix 4:

Competitive economy: General Case

The commodity production function can be written as

$$y_c = f(y_s, K) \tag{A.1}$$

Profit function for commodity sector

$$\pi_c = 1.y_c - r(1-\varphi)K - p_s y_s \tag{A.2}$$

The profit maximisation conditions are

$$\frac{d\pi_c}{d\{(1-\varphi)K\}} = 0 \tag{A.3}$$

Or, $A\alpha\{(1-\varphi)K\}^{\alpha-1} y_s^{\ 1-\alpha} - r = 0$

Or, $A\alpha\{(1-\varphi)K\}^{\alpha-1} y_s^{\ 1-\alpha} = r$ (A.4)

Substituting the value of y_s in equation (A.4)

$$A\alpha\{(1-\varphi)K\}^{\alpha-1}\{B(Nh)^{\beta}(\varphi K)^{1-\beta}\}^{1-\alpha} = r$$

Or, $A\alpha(1-\varphi)^{\alpha-1} K^{(\alpha-1)+(1-\beta)(1-\alpha)} B^{1-\alpha} (Nh)^{\beta(1-\alpha)} \varphi^{(1-\beta)(1-\alpha)} = r$

Or, $A\alpha(1-\varphi)^{\alpha-1} B^{1-\alpha} (\frac{K}{Nh})^{-\beta(1-\alpha)} \varphi^{(1-\beta)(1-\alpha)} = r$

Or, $A\alpha(1-\varphi)^{\alpha-1} B^{1-\alpha} k^{-\beta(1-\alpha)} \varphi^{(1-\beta)(1-\alpha)} = r$ (A.5)

$$\frac{d\pi_c}{dy_s} = 0$$

Or, $A\{(1-\varphi)K\}^{\alpha}(1-\alpha)y_s^{\,1-\alpha-1} - p_s = 0$

$$A\{(1-\varphi)K\}^{\alpha}(1-\alpha)y_s^{\,-\alpha} = p_s \qquad (A.6)$$

The service output production function can be written as

$$y_s = f(Nh, K)$$

Profit function for service sector is

$$\pi_s = p_s y_s - (r\varphi K) - (wNh)$$

Or, $\pi_s = p_s B(Nh)^{\beta}(\varphi K)^{1-\beta} - (r\varphi K) - (wNh)$ (A.7)

The profit maximisation conditions are

$$\frac{d\pi_s}{d(\varphi K)} = 0$$

Or, $p_s B(Nh)^{\beta}(1-\beta)(\varphi K)^{-\beta} - r = 0$

Or, $p_s B(Nh)^{\beta}(1-\beta)(\varphi K)^{-\beta} = r$ (A.8)

$$\frac{d\pi_s}{d(Nh)} = 0$$

Or, $p_s B(Nh)^{\beta-1}\beta(\varphi K)^{1-\beta} - w = 0$

Or, $p_s B(Nh)^{\beta-1}\beta(\varphi K)^{1-\beta} = w$ (A.9)

The Hamiltonian function can be formulated as

$$H = \frac{c^{1-\sigma}}{(1-\sigma)}N + \theta[(1-\tau)(rK + wNh) - cN] \qquad (A.10)$$

Here c is the decision variable and K is the state variable.

$$\frac{dH}{dc} = 0$$

Or, $c^{-\sigma} = \theta$ (A.11)

Taking logarithm both sides and differentiating with respect to time

$$-\sigma\frac{\dot{c}}{c}=\frac{\dot{\theta}}{\theta}$$

Or, $-\sigma\gamma_c=\frac{\dot{\theta}}{\theta}$ (A.12)

The co-state equation can be written as

$$\dot{\theta}=\rho\theta-\frac{dH}{dK} \quad \text{(A.13)}$$

Here $\frac{dH}{dK}=\theta(1-\tau)r$ (A.14)

Substituting this value from (A.14) into (A.13)

$$\dot{\theta}=\rho\theta-\theta(1-\tau)r$$

Or, $\frac{\dot{\theta}}{\theta}=\rho-(1-\tau)r$ (A.15)

Substituting the value from (A.12) we get

$$-\sigma\gamma_c=\rho-(1-\tau)r$$

Or, $\gamma_c=\frac{(1-\tau)r-\rho}{\sigma}$ (A.16)

Substituting the value of r from equation (A.5) in above equation we get

$$\gamma_c=\frac{(1-\tau)A\alpha(1-\varphi)^{\alpha-1}B^{1-\alpha}k^{-\beta(1-\alpha)}\varphi^{(1-\beta)(1-\alpha)}-\rho}{\sigma} \quad (A.16')$$

From equations (A.5) and (A.8)

$$A\alpha\{(1-\varphi)K\}^{\alpha-1}y_s^{1-\alpha}=p_sB(Nh)^{\beta}(1-\beta)(\varphi K)^{-\beta}$$

Or, $A\alpha\{(1-\varphi)K\}^{\alpha-1}\{B(Nh)^{\beta}(\varphi K)^{1-\beta}\}^{1-\alpha}=p_sB(Nh)^{\beta}(1-\beta)(\varphi K)^{-\beta}$

Or, $A\alpha\{(1-\varphi)K\}^{\alpha-1}B^{-\alpha}(Nh)^{\beta(1-\alpha)-\beta}(\varphi K)^{(1-\beta)(1-\alpha)+\beta}=p_s(1-\beta)$ (A.17)

From equation (A.6) we got

$$A\{(1-\varphi)K\}^{\alpha}(1-\alpha)y_s^{1-\alpha}=p_s$$

And substituting the value of y_s in above expression we have

$$A\{(1-\varphi)K\}^{\alpha}(1-\alpha)\{B(Nh)^{\beta}(\varphi K)^{1-\beta}\}^{-\alpha} = p_s \quad \text{(A.18)}$$

Or, $A(1-\varphi)^{\alpha}(1-\alpha)B^{-\alpha}(\frac{K}{Nh})^{\beta\alpha}\varphi^{-\alpha(1-\beta)} = p_s$

Or, $A(1-\varphi)^{\alpha}(1-\alpha)B^{-\alpha}k^{\beta\alpha}\varphi^{-\alpha(1-\beta)} = p_s$ (A.19)

Substituting the value of p_s from equation (A.19) into equation (A.17)

$$\frac{(1-\hat{\varphi})}{\hat{\varphi}} = \frac{\alpha}{(1-\alpha)(1-\beta)}$$

Or, $\hat{\varphi} = \frac{(1-\alpha)(1-\beta)}{1-\beta(1-\alpha)}$

Now, $\gamma_h^{comp} = \eta\tau A\{(1-\hat{\varphi})\}^{\alpha} B^{1-\alpha}(\hat{\varphi})^{(1-\beta)(1-\alpha)} k^{1-\beta(1-\alpha)}$ (A.20)

$$\gamma_c^{comp} = \frac{(1-\tau)\alpha A(1-\hat{\varphi})^{\alpha} B^{1-\alpha}\hat{\varphi}^{(1-\beta)(1-\alpha)}k^{-\beta(1-\alpha)} - \rho}{\sigma} \quad \text{(A.21)}$$

Equating equation (A.20) and (A.21) we get

$$\sigma\eta\tau A\{(1-\hat{\varphi})\}^{\alpha} B^{1-\alpha}(\hat{\varphi})^{(1-\beta)(1-\alpha)} k^{1-\beta(1-\alpha)} + \rho = (1-\tau)\alpha A(1-\hat{\varphi})^{\alpha} B^{1-\alpha}\hat{\varphi}^{(1-\beta)(1-\alpha)}k^{-\beta(1-\alpha)} \quad \text{(A.22)}$$

Let $f(k) = L.H.S$ of equation (A.22), where $f' > 0$. $f(k)$

$g(k) = R.H.S$ of equation (A.22), where $g' < 0$. $g(k)$

Diagrammatically, it can be shown that there exists unique value of k in terms of τ and other parameters.